Instantaneous Frequency

- Sinusoidal signal:

$$g(t) = A\cos(2\pi \boldsymbol{f_0} t + \phi)$$

  - Frequency $= \boldsymbol{f_0}$
- Generalized sinusoidal signal:

$$g(t) = A\cos\big(\theta(t)\big)$$

  - Frequency = ?
    - Observation: Frequency value may vary as a function of time.
      - "**instantaneous frequency**"
- Why do we need to find the instantaneous frequency?
  - Analyze Doppler effect (or Doppler shift)
  - Implement **frequency modulation** (**FM**)
    - where the instantaneous frequency will follow the message $m(t)$.

Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

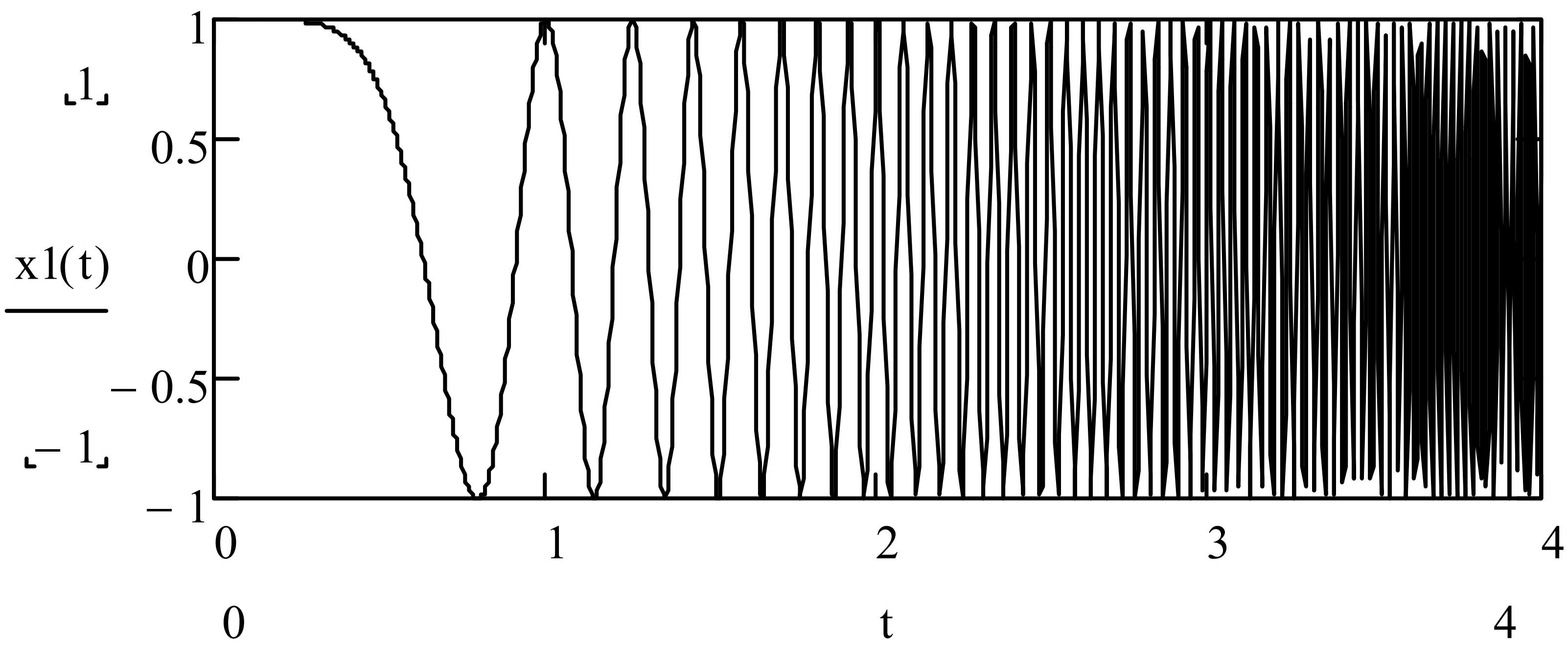

Figure 38

At t = 2, frequency = ?

Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

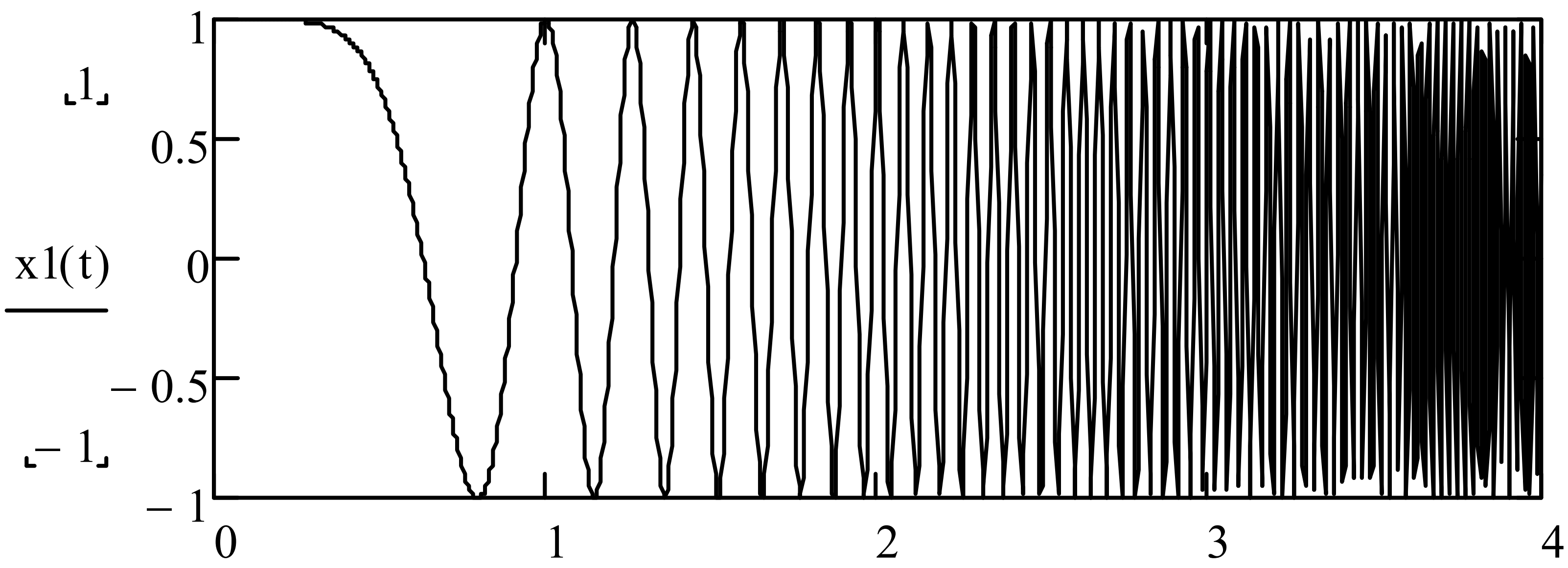

Figure 38

By matching the terms with $\cos(2\pi \boldsymbol{f}_0 t)$, you may guess that $\boldsymbol{f}(\boldsymbol{t}) = t^2$.

At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frequency

$$x_1(t) = \cos(2\pi t^2 t)$$

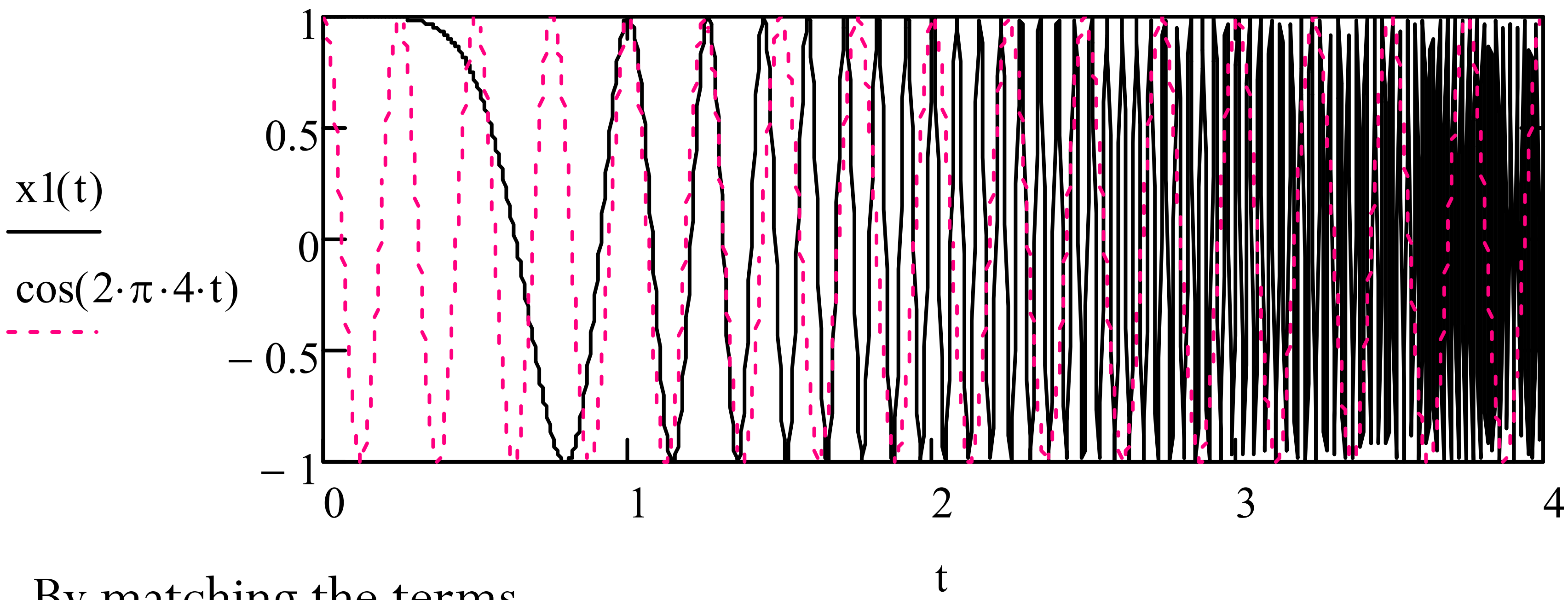

Figure 38

By matching the terms with $\cos(2\pi f_0 t)$, you may guess that $f(t) = t^2$.

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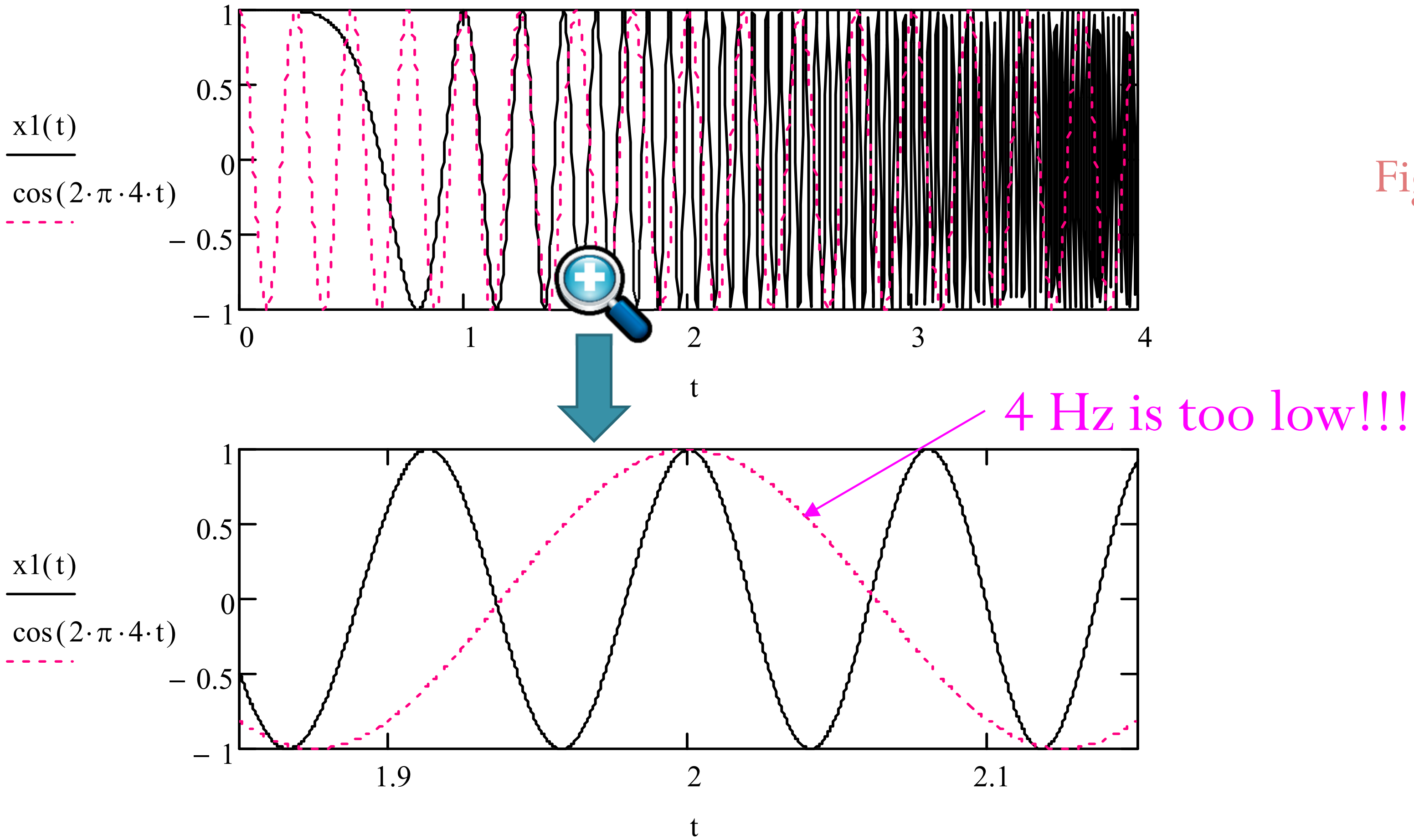
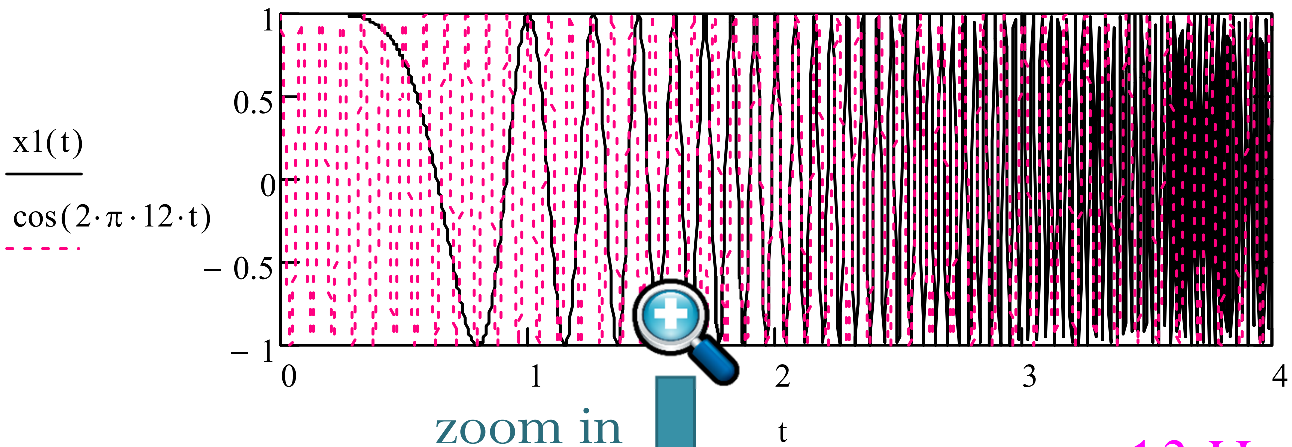


Figure 38

Instantaneous Frequency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



zoom in

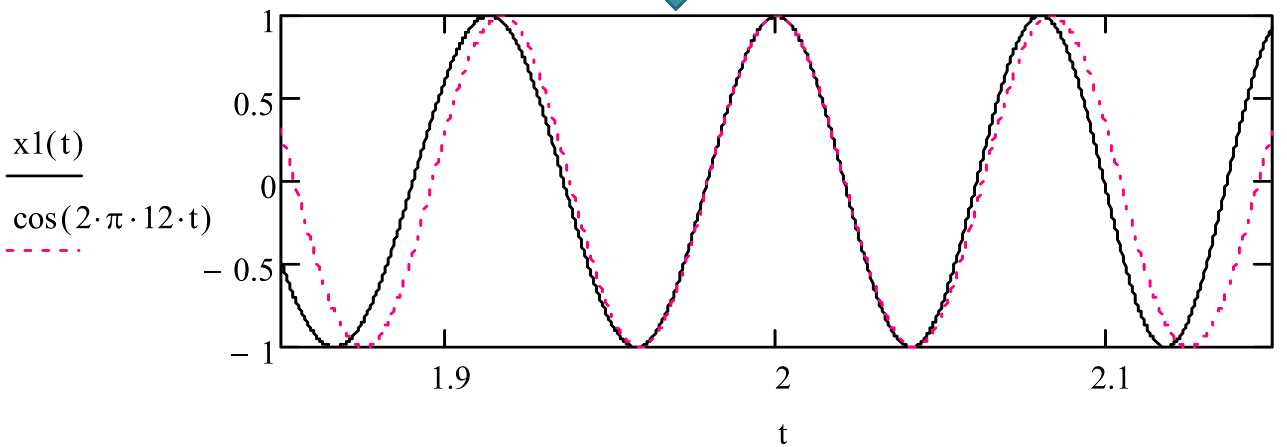


Figure 38

12 Hz fits better.

12 Hz?

Instantaneous Frequency

- Sinusoidal signal:

$$g(t) = A\cos(2\pi \boldsymbol{f_0} t + \phi)$$

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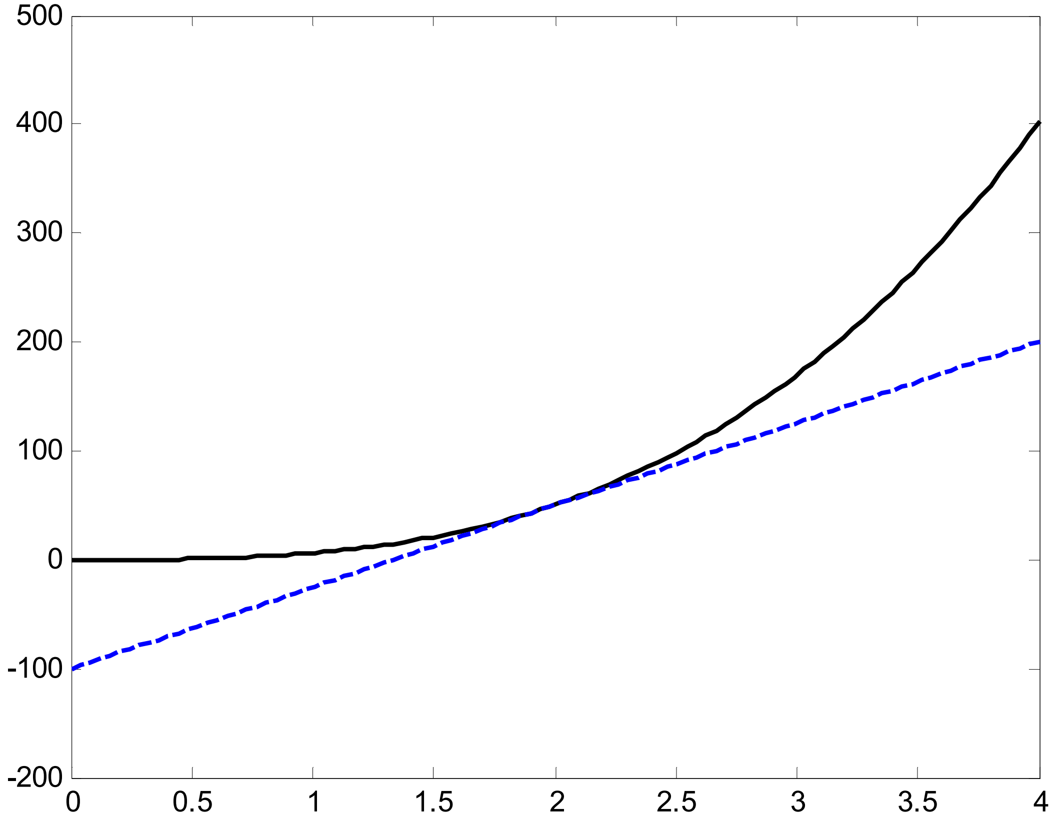
- Generalized sinusoidal signal:

$$g(t) = A\cos\big(\theta(t)\big)$$

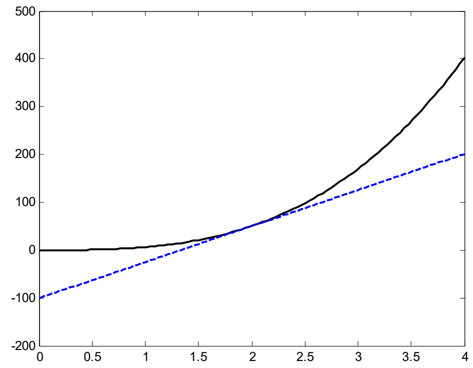
  - Frequency = ?
    - Observation: Frequency value may vary as a function of time.
      - "**instantaneous frequency**"
- Why do we need to find the instantaneous frequency?
  - Analyze Doppler effect (or Doppler shift)
  - Implement **frequency modulation** (**FM**)
    - where the instantaneous frequency will follow the message $m(t)$.

First-order (straight-line) approximation/linearization

- How does the formula $f(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t)$ work?
- Technique from Calculus: first-order (tangent-line) approximation/linearization



First-order (straight-line) approximation/linearization



- How does the formula $f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$ work?
- Technique from Calculus: first-order (tangent-line) approximation/linearization
  - When we consider a function $\theta(t)$ near a particular time, say, $t = t_0$, the value of the function is approximately

$$\theta(t) \approx \underbrace{\theta'(t_0)}_{\text{slope}}(t - t_0) + \theta(t_0) = \underbrace{\theta'(t_0)}_{\text{slope}}t + \underbrace{\theta(t_0) - t_0\theta'(t_0)}_{\text{constant}}$$

  - Therefore, near $t = t_0$,

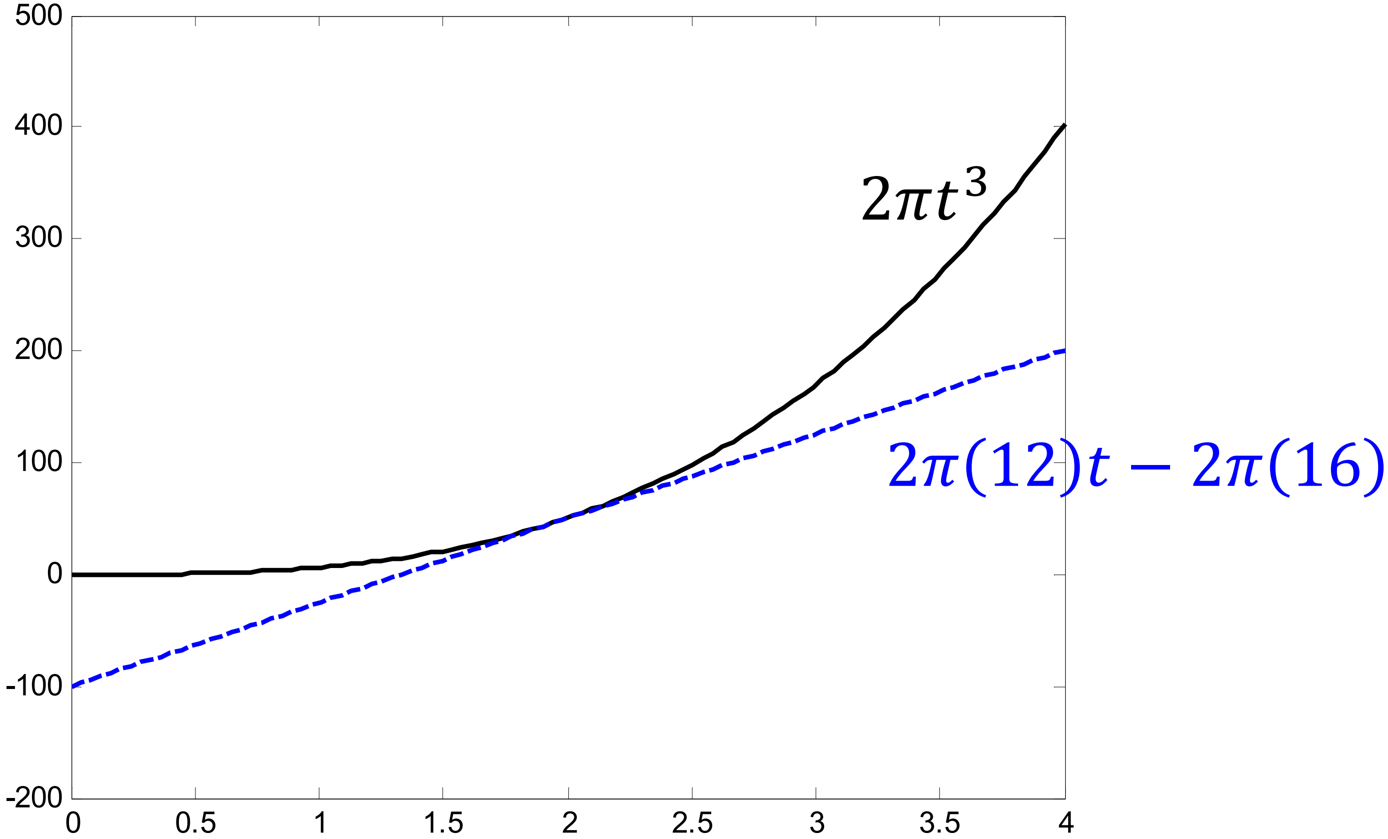
$$\cos(\theta(t)) \approx \cos(\theta'(t_0)t + \theta(t_0) - t_0\theta'(t_0))$$

  - Now, we can directly compare the terms with $\cos(2\pi f_0 t + \phi)$.

First-order (straight-line) approximation/linearization

- For example, for t near $t = 2$,

$$2\pi t^3 \approx 2\pi\left(3t^2\right)\Big|_{t=2}(t-2) + 2\pi t^3\Big|_{t=2} = 2\pi(12)t - 2\pi(16)$$



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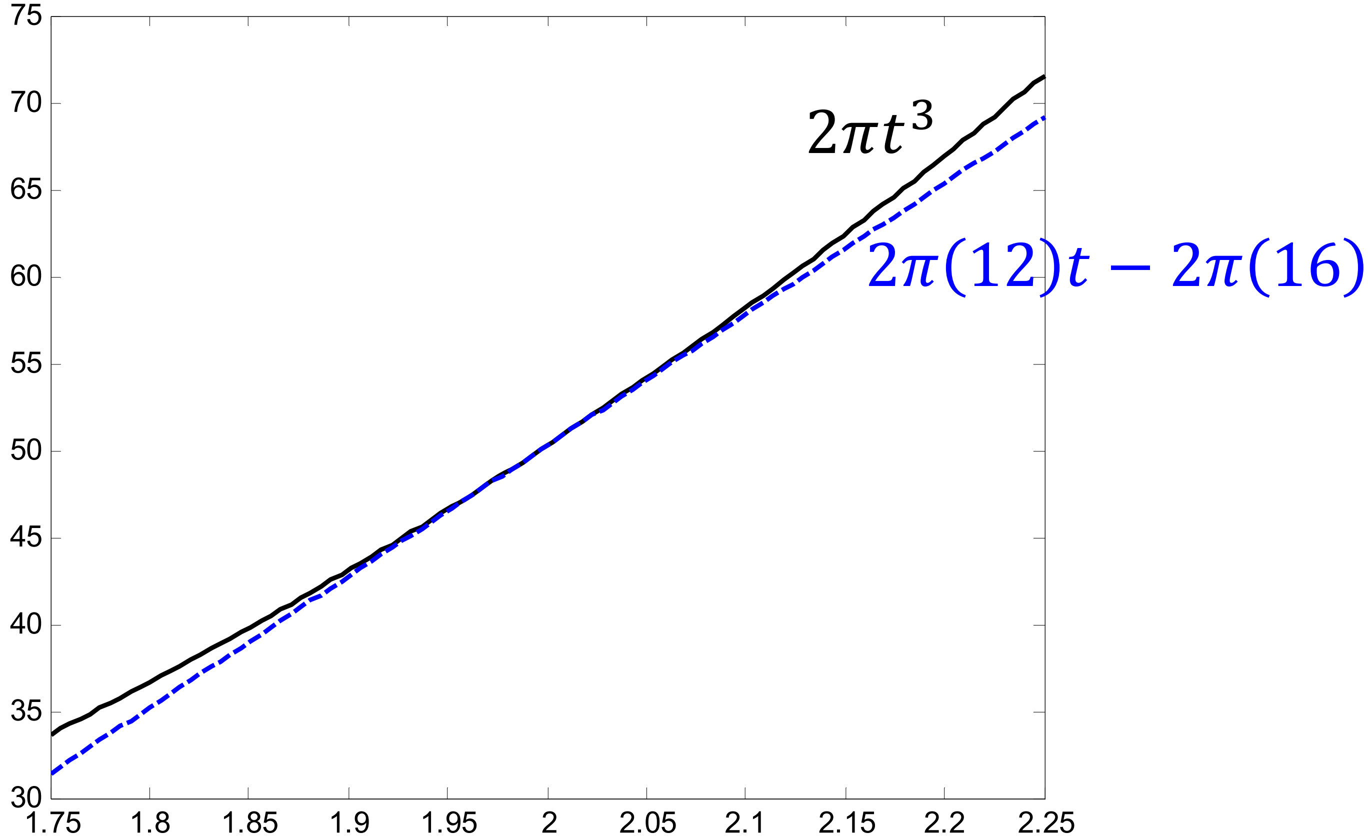

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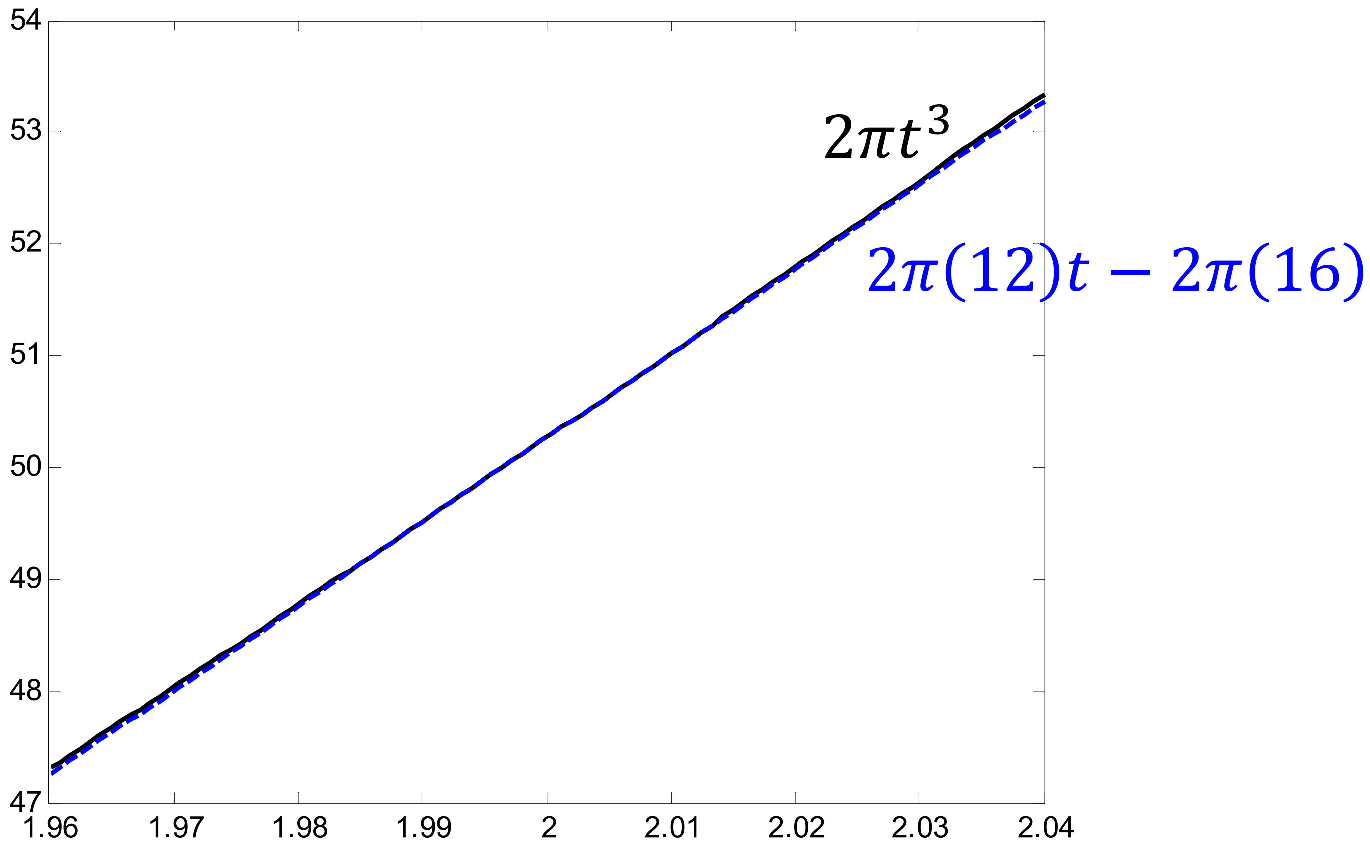

Same idea

- Suppose we want to find $\sqrt{15.9}$.
- Let $g(x) = \sqrt{x}$.
  - Note that $\frac{d}{dx} g(x) = \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.
- Approximation: $g(x) \approx g'(x_0)(x - x_0) + g(x_0)$
- 15.9 is near 16.
- $\sqrt{15.9} = g(15.9)$
  - $\approx g'(16)(15.9 - 16) + g(16)$
  - $= \frac{1}{2\sqrt{16}}(-0.1) + \sqrt{16} = -\frac{0.1}{8} + 4 = 3.9875$
- MATLAB:

```
>> sqrt(15.9)
ans =
   3.987480407475377
```